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# Heterodyne interferometric measurement of the thermo-optic coefficients of potassium niobate

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A heterodyne interferometric technique is applied to determine the thermo-optic coefficients of a potassium niobate crystal with a GaAlAs diode laser. The heterodyne beats signal, resulting from the beating of the carrier and two side-band frequencies, is related to the phase shift difference between the carrier and the side-bands. This phase shift difference is determined with a phase sensitive detection technique and is found to be temperature dependent. From the temperature dependence, the thermo-optic coefficients of the potassium niobate crystal along the  $b$  and  $c$  crystallographic axes are determined to be  $-4.5 \times 10^{-5}$  and  $6.7 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$ , respectively.

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## INTRODUCTION

The thermo-optic coefficient ( $dn/dT$ ) is an important parameter for the nonlinear crystals used in high power harmonic generation devices.<sup>1-3</sup> This is because even only small optical absorption at fundamental and/or harmonic wavelengths can cause nonuniform heating. Temperature gradients thus build up inside the crystals and spatial variation of refractive index is induced through  $dn/dT$ . As a result, phase matching is adversely affected, and the harmonic generation efficiency becomes very low.<sup>1-5</sup> In addition,  $dn/dT$  at fundamental frequency relative to that at second harmonic frequency, known as temperature tuning coefficient, is an important parameter for noncritical phase matching in second harmonic generation.<sup>6-8</sup> This coefficient and the birefringence of the crystal determine whether or not a crystal can be temperature tuned to satisfy phase matching condition.  $dn/dT$  is therefore a very important property of nonlinear crystals in general.

Potassium niobate ( $\text{KNbO}_3$ ) has been shown to be an efficient nonlinear crystal for the upconversion of near infrared (IR) radiation such as the extra-cavity frequency doubling of a GaAlAs diode laser,<sup>9,10</sup> and the sum frequency mixing of a titanium:sapphire ( $\text{Ti:Al}_2\text{O}_3$ ) laser with a diode pumped Nd:YVO<sub>4</sub> laser,<sup>11</sup> and a GaAlAs diode laser with an InGaAs diode laser.<sup>12</sup> The blue lasers thus produced are useful since their powers are great enough for practical applications such as optical data storage, undersea communication, full color display, and lithography. In the frequency mixing

of diode lasers with the  $\text{KNbO}_3$  crystal, noncritical phase matching is mostly used because of the crystal's suitable temperature tuning coefficient and birefringence. In order to predict the phase matching temperature at a given wavelength, the  $dn/dT$  value and the principal refractive indices of  $\text{KNbO}_3$  must be known to a reasonable accuracy. The former is addressed in this article, making use of a novel heterodyne interferometric technique.

The thermo-optic coefficient of a nonlinear crystal is commonly determined by the method of the angle of minimum deviation (AMD).<sup>2,3</sup> The  $dn/dT$  values determined with this method are actually averaged over finite temperature increments. It can thus be applied only under the conditions of constant  $dn/dT$  and constant thermal expansion coefficient. Furthermore, because this technique gives a value of refractive index accurate only to the fourth digit after the decimal point, the  $dn/dT$  value is also limited to the same accuracy. However, the  $dn/dT$  values of most materials are on the order of  $10^{-5}$ – $10^{-6}$ . Thus the  $dn/dT$  values determined with AMD are not accurate enough. We have previously introduced a more accurate technique, using the Fabry–Pérot interferometer, for the determination of the nonlinear crystal  $dn/dT$ , not just  $\Delta n/\Delta T$ .<sup>13</sup> Recently, a heterodyne type Mach–Zehnder interferometer has been used to measure the thermo-optic coefficients of quartz retardation plates,<sup>14</sup> with an accuracy five to six digits after the decimal point. In contrast to the two-path Mach–Zehnder interferometer, we demonstrate in this article a novel heterodyne one-path Fabry–Pérot interferometric technique for measuring the  $dn/dT$  values of nonlinear crystals with comparable accuracy.

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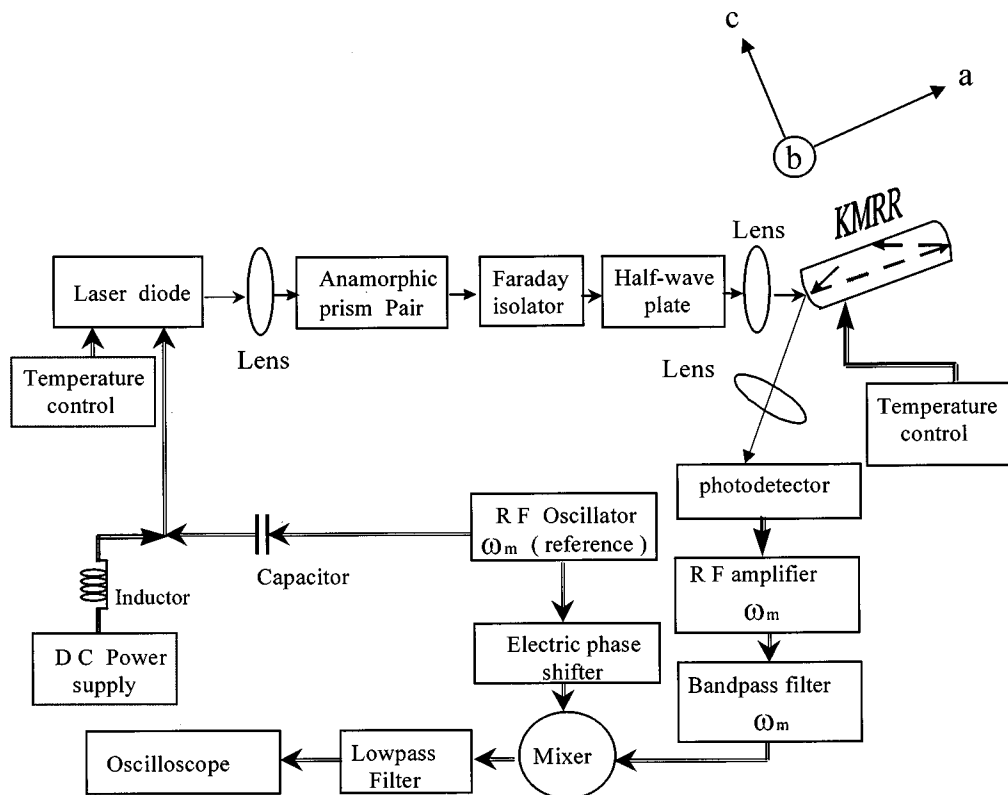


FIG. 1. The experimental setup for the heterodyne interferometric measurement of thermo-optic coefficients of the  $\text{KNbO}_3$  monolithic ring resonator (KMRR). The direction of crystallographic axes of the  $\text{KNbO}_3$  crystal are illustrated at the top right: (Single line arrows) optical; (double line arrows) electrical.

## EXPERIMENT

The experimental set up is shown in Fig. 1. The light source used was a distributed Bragg reflector GaAlAs strained layer laser diode (SDL 5712-H1) which provided a stable output wavelength at 852 nm with a 3 MHz linewidth. The laser was driven by a dc current and simultaneously modulated by a very weak radio frequency (rf) current,  $\omega_m = 280$  MHz, from the rf oscillator. Consequently, the diode laser spectrum contained three frequencies; the carrier ( $\omega_0$ ) and two side bands ( $\omega_0 \pm \omega_m$ ). The laser with these three frequencies was first collimated with an  $f = 7.5$  mm lens and aligned to go through an anamorphic prism pair so that its transverse intensity distribution became round. The laser was then sent through a Faraday isolator, which blocked the light reflected from the optics down stream, and a half-wave plate, for laser polarization rotation. The lens before the thermo-electrically cooled  $\text{KNbO}_3$  monolithic ring resonator (KMRR) served to focus the laser beam, whereas the other one served to couple to the electric system downstream.

The KMRR was fabricated with sides parallel to the three principal crystallographic axes (upper right, Fig. 1). It had the dimensions of  $7 \times 3 \times 2$  mm ( $a \times b \times c$ ). Both curved surfaces had a 5 mm radius of curvature and a high reflectivity for 860 nm, and hence formed a Fabry-Pérot resonator. The incident laser was arranged to circulate inside the crystal in an isosceles triangle with the larger side parallel to the crystallographic  $a$  axis. The polarization of the incident laser was set along either the crystallographic  $b$  or  $c$  axis depending on whether  $\partial n_b / \partial T$  or  $\partial n_c / \partial T$  was to be measured.

The photodetector detected the signal arising from the heterodyne beating of the multi-reflected laser beam from the KMRR at the carrier and side-band frequencies. A rf amplifier amplified the signal from the photodetector, which is fast enough to follow the  $\omega_m$  signal oscillation although not fast enough to follow the carrier laser oscillation. The amplified rf signal at  $\omega_m$  was allowed to pass a bandpass filter and then mixed with the reference signal in a rf mixer. The dc output of the mixer, which was related to the phase shift difference between the carrier and the side-bands, was obtained through a lowpass filter. The dc signal was monitored and recorded by an oscilloscope while the temperature in the KMRR was being changed continuously.

## RESULTS AND DISCUSSION

If the injection modulation current of the diode laser is weak, the spectrum of the diode laser contains a strong carrier at  $\omega_0$  and two weak side bands at  $\omega_0 \pm \omega_m$ ; higher order side bands are negligible. The optical field of the diode laser output is approximated with<sup>15</sup>

$$E(t) \approx E_0 \{ J_0(\beta) e^{i\omega_0 t} + J_1(\beta) e^{i(\omega_0 + \omega_m)t} + J_{-1}(\beta) e^{i(\omega_0 - \omega_m)t} \}, \quad (1)$$

where  $E_0$  is the amplitude of the optical field and  $J_n(\beta)$  ( $n = 0, \pm 1$ ) are the Bessel functions.  $\beta$  is the frequency modulation index, which is experimentally controlled at a very low level. As shown in Fig. 1, the laser optical field is arranged to circulate inside the KMRR in an isosceles triangle

through multiple reflection. The multiple-reflected optical field leaving the front surface of the KMRR is then

$$E_r(t) \approx E_0 [T_0(\omega_0)J_0(\beta)e^{i\omega_0 t} + T_1(\omega_0 + \omega_m)J_1(\beta)e^{i(\omega_0 + \omega_m)t} + T_{-1}(\omega_0 - \omega_m)J_{-1}(\beta)e^{i(\omega_0 - \omega_m)t}]. \quad (2)$$

Let  $\omega_n = \omega_0 \pm n\omega_m$  and  $n = 0, \pm 1$ . The complex reflection functions  $T_n(\omega_n)$  then becomes

$$T_n(\omega_n) = \exp(-\delta_n - i\varphi_n), \quad (3)$$

where  $\delta_n$  is the amplitude attenuation and  $\varphi_n$  is the optical phase shift at  $\omega_n$ . With  $\beta \ll 1$  and Eqs. (3) and (2) can be simplified to

$$E_r(t) \approx E_0 \left( T_0(\omega_0)e^{i\omega_0 t} + T_1(\omega_1)\frac{\beta}{2}e^{i\omega_1 t} + T_{-1}(\omega_{-1})\frac{-\beta}{2}e^{i\omega_{-1} t} \right). \quad (4)$$

Since a rf amplifier and a band pass filter are used to select only the  $\omega_m$  frequency, the terms needed to be considered in the calculation of the light intensity  $I(t)$  detected by the photodetector are

$$I(t) \approx E_0^2 \beta e^{-\delta_0} \{ [e^{-\delta_1} \cos(\varphi_0 - \varphi_1) - e^{-\delta_{-1}} \cos(\varphi_{-1} - \varphi_0)] \cos \omega_m t + [e^{-\delta_{-1}} \sin(\varphi_{-1} - \varphi_0) - e^{-\delta_1} \sin(\varphi_0 - \varphi_1)] \sin \omega_m t \}. \quad (5)$$

The phase of the rf reference signal is set to be  $\sin \omega_m t$  by adjusting the electric phase shifter. The detected signal output  $V(t)$  from the mixer is then

$$V(t) \propto E_0^2 \beta e^{-\delta_0} \{ [e^{-\delta_1} \cos(\varphi_0 - \varphi_1) - e^{-\delta_{-1}} \cos(\varphi_{-1} - \varphi_0)] \cos \omega_m t \sin \omega_m t + [e^{-\delta_{-1}} \sin(\varphi_{-1} - \varphi_0) - e^{-\delta_1} \sin(\varphi_0 - \varphi_1)] \sin^2 \omega_m t \}. \quad (6)$$

Since a low pass filter is used after the mixer to block the ac electrical signal, only the dc signal  $V(\varphi_n, \delta_n)$  in Eq. (6) is measured by the oscilloscope,

$$V(\varphi_n, \delta_n) \propto \frac{1}{2} E_0^2 e^{-\delta_0} \beta [e^{-\delta_{-1}} \sin(\varphi_{-1} - \varphi_0) - e^{-\delta_1} \sin(\varphi_0 - \varphi_1)]. \quad (7)$$

The dc signal  $V(\varphi_n, \delta_n)$  is thus related to the optical phase shift difference between the carrier and the two side bands in the KNbO<sub>3</sub> resonator, i.e.,  $\varphi_{-1} - \varphi_0$  and  $\varphi_0 - \varphi_1$ . This is true even if the phase shift of the rf reference signal is adjusted to be  $\cos \omega_m t$ , for which the first term of Eq. (6) should be considered instead of the second term. When the resonator's resonance frequency is temperature-tuned to agree with the carrier frequency, we have  $\delta_{-1} = \delta_1$  and  $\sin(\varphi_{-1} - \varphi_0) = \sin(\varphi_0 - \varphi_1)$ . Then, by Eq. (7) the dc electrical signal is zero.

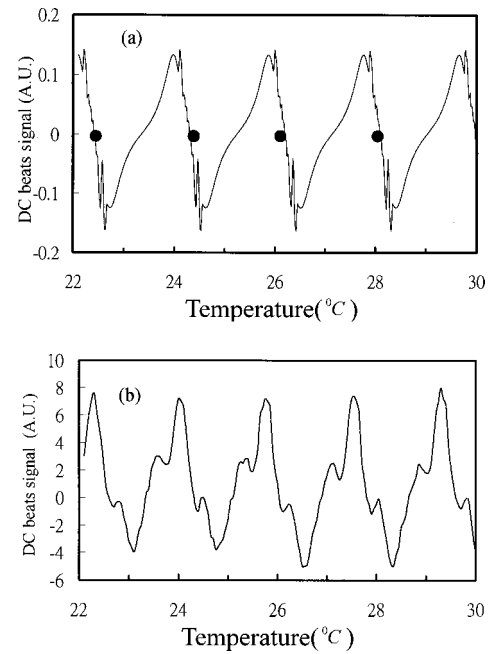


FIG. 2. The dc beats signal vs temperature for the laser polarized along the crystallographic  $b$  axis: (a) computer simulation result, (b) experimental measurement result. At the dots in (a) with zero dc voltage, carrier frequency equals resonator's resonance frequency.

After multiple-reflection in the KMRR, the laser of frequency  $\omega_n$  suffers amplitude attenuation  $\delta_n$  and optical phase shift  $\varphi_n$ ,

$$\exp(-\delta_n) = \frac{r[(1 - \cos \theta_n)^2 + (\sin \theta_n)^2]^{1/2}}{[(1 - r^2 \cos \theta_n)^2 + (r \sin \theta_n)^2]^{1/2}} \quad (8)$$

and

$$\varphi_n = \tan^{-1} \left[ \frac{\sin \theta_n}{1 - \cos \theta_n} \right] + \tan^{-1} \left[ \frac{r^2 \sin \theta_n}{1 - r^2 \cos \theta_n} \right], \quad (9)$$

respectively. Here  $r$  is the amplitude reflection coefficient and  $\theta_n$  is the optical phase shift of the diode laser at  $\omega_n$  after one round trip in the KNbO<sub>3</sub> resonator,

$$\theta_n = \frac{n_{\omega_n} L \omega_n}{c}, \quad (10)$$

where  $L$  is the round-trip cavity length of the resonator at room temperature, and  $c$  is the speed of light in vacuum.  $n_{\omega_n}$  is the index of refraction of the KNbO<sub>3</sub> crystal at  $\omega_n$ . The  $n_{\omega_n}$  value at a given temperature can be calculated from the Zysset's temperature dependent Sellmeier equation for KNbO<sub>3</sub>.<sup>16</sup> With this  $n_{\omega_n}$  and holding  $L$ ,  $c$ , and  $\omega_n$  fixed,  $\theta_n$  for the chosen temperature is calculated from Eq. (10). Equations (8) and (9) are then used to calculate amplitude attenuation  $\delta_n$  and optical phase shift  $\varphi_n$ , respectively, with a given  $r$ . Use is finally made of Eq. (7) for a calculation of the dc beats signal. Repeating the same procedure for various temperatures, the dc beats signal as a function of temperature is obtained and presented below.

Figure 2(a) shows the simulated dc beats signal as a function of temperature without considering thermal expansion effect on  $L$ . In this simulation, it is assumed that laser

polarization is along the crystallographic  $b$  axis,  $r=0.7$ ,  $\beta=0.1$ , and  $E_0=1.0$ . As illustrated in this figure, the simulated dc signal oscillates periodically as the crystal temperature changes. The dotted points at zero dc voltage indicate the temperatures at which the resonance frequency of the  $\text{KNbO}_3$  resonator become coincident with the laser carrier frequency. The temperature change  $\Delta T$  needed to go from one dot point to its neighboring point is found to be  $1.83^\circ\text{C}$ . Note that a more realistic  $r$  of 0.9 leads to identical  $\Delta T$  except for a slightly different fine structure. Figure 2(b) shows the experimental dc beats signal. It also oscillates with crystal temperature. The period ( $\Delta T$ ) is found to be  $1.79^\circ\text{C}$ , which is slightly smaller than the simulation result. This small discrepancy may be due to the temperature dependence of  $L$ , not considered in simulation. The  $\Delta T$  satisfies the following equation;

$$\frac{dn}{dT} \Delta TL + n\alpha L \Delta T = \lambda, \quad (11)$$

where  $n$  is the refractive index of the  $\text{KNbO}_3$  crystal at  $\omega_0$ , and is either  $n_b$  or  $n_c$  depending on the laser polarization. It is calculated from the Zysset's Sellmeier equation.  $\lambda$  is the carrier wavelength of the diode laser, and  $\alpha$  is the thermal expansion coefficient along the crystallographic  $a$  axis. Note that thermal expansion along the crystallographic  $c$  axis is neglected since the angle between the crystallographic  $a$  axis and the two equal sides of the isosceles triangle is small. For the same reason, the electric field component along the  $a$  axis is neglected when considering the propagation of the  $c$ -polarized laser along the equal sides of the isosceles triangle. With  $\alpha=5.0 \times 10^{-6}^\circ\text{C}^{-1}$ ,  $\Delta T=1.79^\circ\text{C}$ , and  $n_b=2.278$  (calculated from the Zysset's Sellmeier equation),  $dn_b/dT$  is determined to be  $-4.5 \times 10^{-5}^\circ\text{C}^{-1}$  from Eq. (11). This result is very close to the previously reported values.<sup>13,17</sup> The negative sign of  $dn_b/dT$  is adopted from our previous result.<sup>13</sup> Although the Zysset's Sellmeier equation, which gives a value of refractive index accurate only to the fourth digit after the decimal point, is not expected to lead to a  $dn/dT$  with such accuracy the  $\Delta T$  values calculated from Zysset's Sellmeier equation agrees reasonably well with the experimental value (see above). It appears then that the Zysset's Sellmeier equation is accurate enough for the  $b$  polarization at least, for the determination of  $dn/dT$  in the temperature range from 22 to  $30^\circ\text{C}$ .

Figure 3(a) is the computer simulation result of the dc beats signal as a function of temperature for laser polarization along the crystallographic  $c$  axis. As in the  $b$  polarization case, the dc beats signal oscillates periodically as crystal temperature increases. However, a smaller period of  $\Delta T=0.62^\circ\text{C}$  is found. The dots in this figure have the same meaning as in the  $b$  polarization case. Shown in Fig. 3(b) is the experimental dc beats signal. The experimental result is similar to the simulation one except for a longer period.  $\Delta T=0.82^\circ\text{C}$ ; the experimental  $\Delta T$  is slightly smaller in the  $b$  polarization case. The large  $\Delta T$  discrepancy in this case cannot be explained by the thermal expansion factor alone. The answer perhaps lies in a finding with the Zysset's Sellmeier equation, i.e., the temperature measurement inaccuracy associated with  $n_c$  is much larger than that with  $n_b$ .<sup>16</sup>

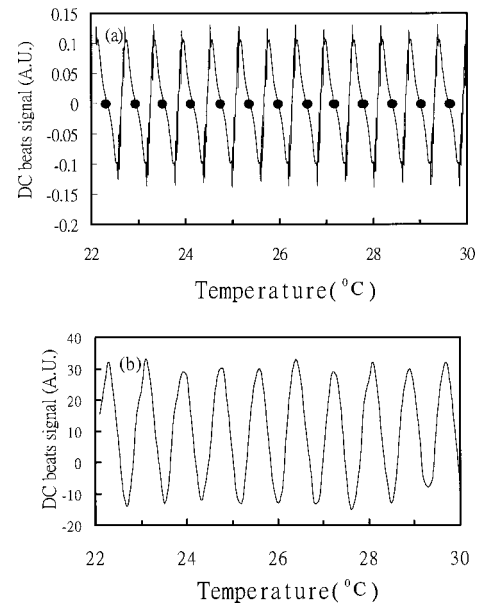


FIG. 3. The dc beats signal vs temperature for the laser polarized along the crystallographic  $c$  axis: (a) computer simulation result, (b) experimental measurement result. At the dots in (a) with zero dc voltage, carrier frequency equals resonator's resonance frequency.

Thus, we resort to the experimental  $\Delta T=0.82^\circ\text{C}$  for the determination of  $dn_c/dT$  in the temperature range from 22 to  $30^\circ\text{C}$ . The  $dn_c/dT$  value is determined to be  $6.7 \times 10^{-5}^\circ\text{C}^{-1}$  from Eq. (11), with  $\alpha=5.0 \times 10^{-6}^\circ\text{C}^{-1}$ , and  $n_c=2.113$  (calculated from the Zysset's Sellmeier equation). The positive sign of  $dn_c/dT$  is adopted again from our previously reported result.<sup>13</sup> The measured  $dn_c/dT$  value is very close to that in the literature.<sup>13,17</sup>

## CONCLUSION

The  $dn/dT$  of the  $\text{KNbO}_3$  crystal are measured by a novel heterodyne interferometric technique. The dc signal, which is related to the phase shift difference between the carrier and the side band frequencies of the incident laser in the  $\text{KNbO}_3$  crystal, is obtained through a phase sensitive detection technique. From the temperature dependence of the dc signal, the  $dn/dT$  of the  $\text{KNbO}_3$  crystal along the  $b$  and  $c$  crystallographic axes are determined to be  $-4.5 \times 10^{-5}$  and  $6.7 \times 10^{-5}^\circ\text{C}^{-1}$ , respectively.

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